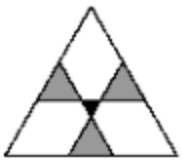
IMC Geometry

1. [IMC 2012 Q16] The diagram shows a large equilateral triangle divided by three straight lines into seven regions. The three grey regions are equilateral triangles with sides of length 5cm and the central black region is an equilateral triangle with sides of length 2cm.
What is the side length of the original large triangle?

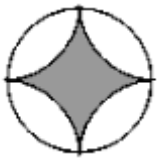


A 18cm B 19cm C 20cm D 21cm E 22cm

2. [IMC 2012 Q18] Peri the winkle starts at the origin and slithers anticlockwise around a semicircle with centre $(4, 0)$. Peri then slides anticlockwise around a second semicircle with centre $(6, 0)$, and finally clockwise around a third semicircle with centre $(3, 0)$. Where does Peri end this expedition?

A $(0, 0)$ B $(1, 0)$ C $(2, 0)$ D $(4, 0)$ E $(6, 0)$

3. [IMC 2012 Q19] The shaded region shown in the diagram is bounded by four arcs, each of the same radius as that of the surrounding circle. What fraction of the surrounding circle is shaded?

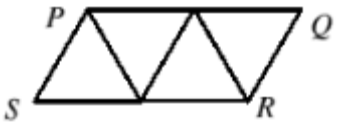


A $\frac{4}{\pi} - 1$ B $1 - \frac{\pi}{4}$ C $\frac{1}{2}$ D $\frac{1}{3}$
E it depends on the radius of the circle

4. [IMC 2012 Q20] A rectangle with area $125cm^2$ has sides in the ratio $4: 5$. What is the perimeter of the rectangle?

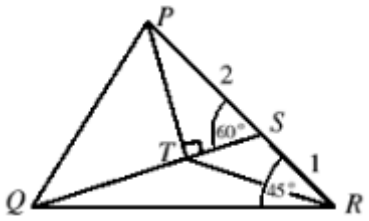
A 18cm B 22.5cm C 36cm D 45cm E 54cm

5. [IMC 2012 Q21] The parallelogram $PQRS$ is formed by joining together four equilateral triangles of side 1 unit, as shown.
What is the length of the diagonal SQ?



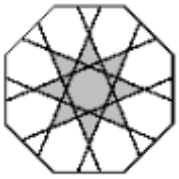
A $\sqrt{7}$ B $\sqrt{8}$ C 3 D $\sqrt{6}$ E $\sqrt{5}$

6. [IMC 2012 Q23] In triangle PQR, $PS = 2$; $SR = 1$; $\angle PRQ = 45°$; T is the foot of the perpendicular from P to QS and $\angle PST = 60°$.
What is the size of $\angle QPR$?



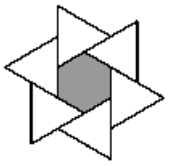
A $45°$ B $60°$ C $75°$ D $90°$
E $105°$

7. [IMC 2012 Q25] The diagram shows a ceramic design by the Catalan architect Antoni Gaudi. It is formed by drawing eight lines connecting points which divide the edges of the outer regular octagon into three equal parts, as shown.
What fraction of the octagon is shaded?



A $1/5$ B $2/9$ C $1/4$ D $3/10$ E $5/16$

8. [IMC 2011 Q18] The diagram contains six equilateral triangles with sides of length 2 and a regular hexagon with sides of length 1.
What fraction of the whole shape is shaded?

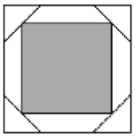


A $\frac{1}{8}$ B $\frac{1}{7}$ C $\frac{1}{6}$ D $\frac{1}{5}$ E $\frac{1}{4}$

9. [IMC 2011 Q19] Harrogate is 23km due north of Leeds, York is 30km due east of Harrogate, Doncaster is 48km due south of York, and Manchester is 70km due west of Doncaster. To the nearest kilometre, how far is it from Leeds to Manchester, as the crow flies?

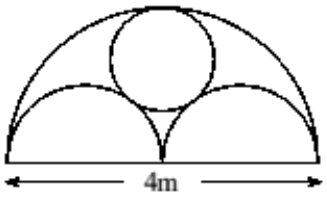
A 38km B 47km C 56km D 65km E 74km

10. [IMC 2011 Q21] A regular octagon is placed inside a square, as shown. The shaded square connects the midpoints of four sides of the octagon.
What fraction of the outer square is shaded?



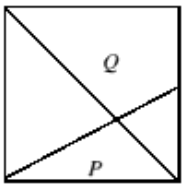
A $\sqrt{2}-1$ B $\frac{1}{2}$ C $\frac{\sqrt{2}+1}{4}$ D $\frac{\sqrt{2}+2}{5}$ E $\frac{3}{4}$

11. [IMC 2011 Q23] A window frame in Salt's Mill consists of two equal semicircles and a circle inside a large semicircle with each touching the other three as shown. The width of the frame is 4m.
What is the radius of the circle, in metres?



A $\frac{2}{3}$ B $\frac{\sqrt{2}}{2}$ C $\frac{3}{4}$ D $2\sqrt{2}-1$ E 1

12. [IMC 2011 Q25] The diagram shows a square, a diagonal and a line joining a vertex to the midpoint of a side. What is the ratio of area P to area Q?



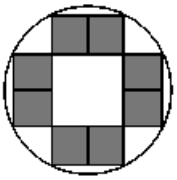
13. [IMC 2010 Q17] Last year Gill's cylindrical 21st birthday cake wasn't big enough to feed all her friends. This year she will double the radius and triple the height. What will be the ratio of the volume of this year's birthday cake to the volume of last year's cake?

A 12:1 B 7:1 C 6:1 D 4:1 E 3:1

14. [IMC 2010 Q19] A snail is at one corner of the top face of a cube with side length 1m. The snail can crawl at a speed of 1m per hour. What proportion of the cube's surface is made up of points which the snail could reach within one hour?

A $\frac{\pi}{16}$ B $\frac{\pi}{8}$ C $\frac{1}{4}$ D $\frac{1}{2}$ E $\frac{\sqrt{3}}{4}$

15. [IMC 2010 Q23] The diagram shows a pattern of eight equal shaded squares inside a circle of area π square units. What is the area (in square units) of the shaded region?

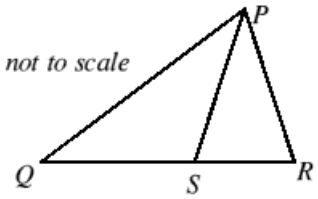


A $1\frac{1}{3}$ B $1\frac{3}{5}$ C $1\frac{2}{3}$ D $1\frac{7}{9}$ E 2

16. [IMC 2010 Q25] Two squares, each of side length $1+\sqrt{2}$ units, overlap. The overlapping region is a regular octagon.
What is the area (in square units) of the octagon?

A $1+\sqrt{2}$ B $1+2\sqrt{2}$ C $2+\sqrt{2}$ D $2+2\sqrt{2}$ E $2+3\sqrt{2}$

17. [IMC 2009 Q17] PQR is a triangle and S is a point on QR.
$QP = QR = 9$cm and $PR = PS = 6$cm.
What is the length SR?

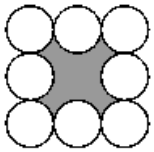


A 1cm B 2cm C 3cm D 4cm E 5cm

18. [IMC 2009 Q20] A square, of side two units, is folded in half to form a triangle. A second fold is made, parallel to the first, so that the apex of this triangle folds onto a point on its base, thereby forming an isosceles trapezium. What is the perimeter of this trapezium?

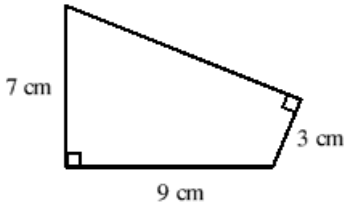
A $4+\sqrt{2}$ B $4+2\sqrt{2}$ C $3+2\sqrt{2}$ D $2+3\sqrt{2}$ E 5

19. [IMC 2008 Q17] The shaded region is bounded by eight equal circles with centres at the corners and midpoints of the sides of a square. The perimeter of the square has length 8. What is the length of the perimeter of the shaded region?



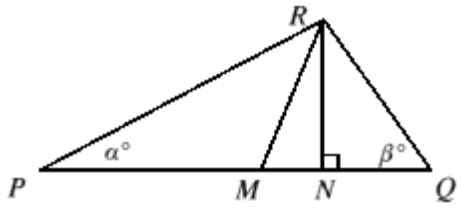
A π B 2π C 8 D 3π E 4π

20. [IMC 2008 Q20] What, in cm^2, is the area of this quadrilateral?



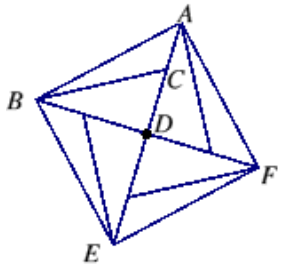
A 48 B 50 C 52 D 54 E 56

21. [IMC 2008 Q21] In triangle PQR, $\angle QPR = \alpha°$ and $\angle PQR = \beta°$, where $\alpha < \beta$. The line RM bisects $\angle PRQ$ and RN is the perpendicular from R to the line PQ. What is the size, in degrees, of $\angle MRN$?



A $\frac{180-(\alpha+\beta)}{2}$ B $\frac{\beta-\alpha}{2}$ C $\frac{\alpha+2\beta}{2}$ D $\frac{360-\alpha-2\beta}{2}$ E $\frac{\alpha+\beta}{2}$

22. [IMC 2008 Q24] The diagram has order 4 rotational symmetry about D. If angle ABC is 15° and the area of $ABEF$ is 24cm^2, what, in cm, is the length of CD?

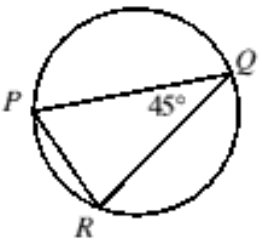


A 1 B $\sqrt{3}$ C 2 D $\sqrt{5}$ E $2\sqrt{3}-1$

23. [IMC 2008 Q25] A garden has the shape of a right-angled triangle with sides of length 30, 40 and 50. A straight fence goes from the corner with the right-angle to a point on the opposite side, dividing the garden into two sections which have the same perimeter. How long is the fence?

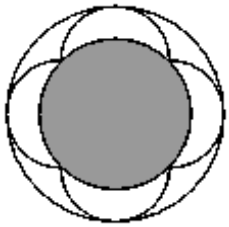
A 25 B $8\sqrt{3}$ C $5\sqrt{11}$ D $5\sqrt{39}$ E $12\sqrt{5}$

24. [IMC 2007 Q20] P, Q, R are points on the circumference of a circle of radius 4cm. $\angle PQR = 45°$. What is the length of chord PR?



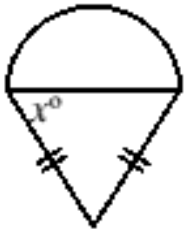
A 4cm B $3\sqrt{3}$cm C $4\sqrt{2}$ cm D 5cm E 6cm

25. [IMC 2007 Q21] The diagram shows two circles and four equal semi-circular arcs. The area of the inner shaded circle is 1. What is the area of the outer circle?



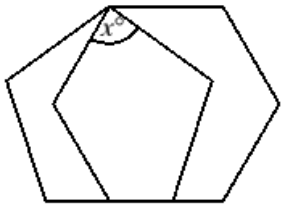
A $\sqrt{2}$ B 2 C $1+\sqrt{2}$ D $\frac{\pi}{2}$ E $\frac{9}{4}$

26. [IMC 2007 Q25] The diagram shows a semi-circle and an isosceles triangle which have equal areas. What is the value of $\tan x°$?



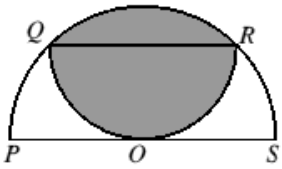
A 1 B $\frac{\sqrt{3}}{2}$ C $\frac{\pi}{\sqrt{3}}$ D $\frac{2}{\pi}$ E $\frac{\pi}{2}$

27. [IMC 2006 Q19] The diagram shows a regular pentagon and a regular hexagon which overlap. What is the value of x?



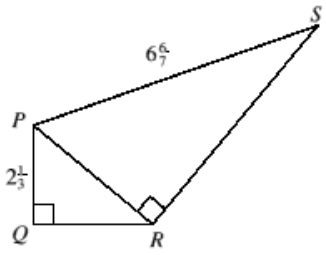
A 82 B 84 C 85 D 87 E 91

28. [IMC 2006 Q21] The diagram shows two semicircular arcs, $PQRS$ and QOR. The diameters, PS and QR, of the two semicircles are parallel; PS is of length 4 and is a tangent to semicircular arc QOR.
What is the area of the shaded region?



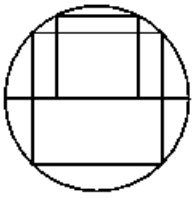
A $2\pi-2$ B 3π C π D 4 E $2\pi-4$

29. [IMC 2006 Q23] In the figure on the right, $PQ=2\frac{1}{3}$, $PS=6\frac{6}{7}$ and $\angle QPR=\angle RPS$. How long is PR?



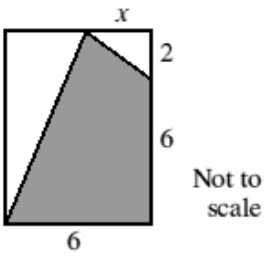
A $3\frac{1}{2}$ B 4 C $4\frac{1}{4}$ D $4\frac{25}{42}$ E 5

30. [IMC 2006 Q24] The diagram shows a square of area x square units inscribed inside a semicircle and a larger square of area y square units inscribed inside a circle. What is the ratio $x:y$?



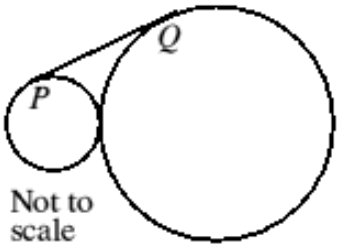
A $1:\sqrt{2}$ B $1:2$ C $2:5$ D $1:3$ E $\sqrt{3}:4$

31. [IMC 2005 Q18] Three-quarters of the area of the rectangle has been shaded. What is the value of x?



A 2 B 2.4 C 3 D 3.6 E 4

32. [IMC 2005 Q21] Two circles with radii 1cm and 4cm touch. The point P is on the smaller circle, Q is on the larger circle and PQ is a tangent to both circles. What is the length of PQ?

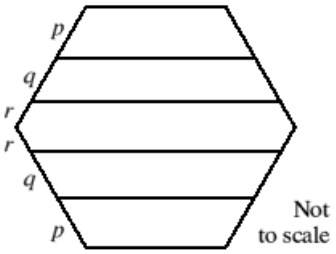


A $\sqrt{17}$cm B 3cm C $2\sqrt{3}$ cm D $3\sqrt{2}$cm E 4cm

33. [IMC 2005 Q23] What is the area (in square units) of the triangle formed by the three lines whose equations are $y - x = 6, x - 2y = 3, x + y = 6$?

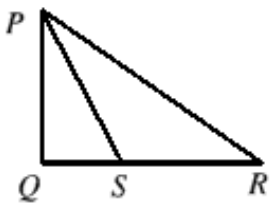
A 55 B 60 C 65 D 70 E 75

34. [IMC 2005 Q25] This regular hexagon has been divided into four trapezia and one hexagon. If each of the five sections has the same perimeter, what is the ratio of the lengths p, q and r?



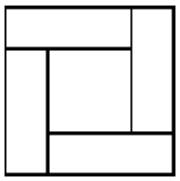
A $8:2:1$ B $12:4:1$ C $9:3:1$ D $6:3:1$ E $9:4:1$

35. [IMC 2004 Q18] In the triangle PQR, there is a right angle at Q and angle QPR is $60°$. The bisector of the angle QPR meets QR at S, as shown.
What is the ratio $QS:SR$?



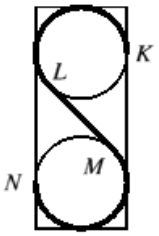
A $1:1$ B $1:\sqrt{2}$ C $1:(3-\sqrt{3})$ D $1:\sqrt{3}$ E $1:2$

36. [IMC 2004 Q21] A square is divided up into four congruent rectangles and a smaller square, as shown. (The diagram is not to scale.) The area of the small square is $\frac{1}{4}$ of the area of the whole square. What is the ratio of the length of a short side of one of the rectangles to the length of a long side?



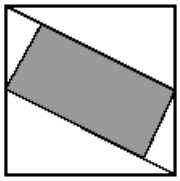
A $1:\sqrt{2}$ B $1:\sqrt{3}$ C $1:2$ D $1:3$ E $1:4$

37. [IMC 2004 Q23] In the diagram, the letter S is made from two arcs, KL and MN, which are each five-eighths of the circumference of a circle of radius 1, and the line segment LM, which is tangent to both circles. At points K and N, common tangents to the two circles touch one of the circles. What is the length LM?



A $\frac{3}{2}$ B $3-\sqrt{2}$ C 2 D $\frac{3\sqrt{2}}{2}$ E $1+\sqrt{2}$

38. [IMC 2004 Q25] The diagram shows a square with two lines from a corner to the middle of an opposite side. The rectangle fits exactly inside these two lines and the square itself. What fraction of the square is occupied by the shaded rectangle?



A $\frac{1}{3}$ B $\frac{2}{5}$ C $\frac{3}{10}$ D $\frac{1}{2}$ E $\frac{3}{8}$

Solutions

1. B
2. C
3. A
4. D
5. A
6. C
7. B
8. D
9. B
10. B
11. A
12. D
13. A
14. B
15. B
16. D
17. D
18. D
19. D
20. A
21. B
22. C
23. E
24. C
25. B
26. E
27. B
28. A
29. B
30. C
31. E
32. E
33. E
34. B
35. E
36. D
37. C
38. B